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Large Numbers Hypothesis. IV. The Cosmological Constant and Quantum Physics

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In standard physics quantum field theory is based on a flat vacuum space-time. This quantum field theory predicts a nonzero cosmological constant. Hence the gravitational field equations do not admit a flat vacuum space-time. This dilemma is resolved using the units covariant gravitational field equations. This paper shows that the field equations admit a flat vacuum space-time with nonzero cosmological constant if and only if the canonical LNH is valid. This allows an interpretation of the LNH phenomena in terms of a time-dependent vacuum state. If this is correct then the cosmological constant must be positive.

1. INTRODUCTION

This is Paper IV in a series seeking to explore the consequences for physics of developing a viable, self-consistent physical theory incorporating Dirac's (1937) large numbers hypothesis (LNH). This paper notes an intimate relationship between the cosmological constant Λ and quantum field theory. In particular, while quantum field theory based on a flat space-time allows one to predict a nonzero value for Λ (DeWitt, 1975; Kirzhnitz and Linde, 1976; Linde, 1979), the gravitational field equations of general relativity do not admit a flat vacuum space-time if $\Lambda \neq 0$.

This dilemma can be resolved in the units covariant theory containing LNH (Adams, 1982). It is shown in Section 3 that the units covariant gravitational field equations admit a flat vacuum solution if and only if the LNH "field" has the canonical LNH (Adams, 1983) form of

$$\varphi = \varphi_0(t_0/t) \tag{1}$$

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where t_0 is related to the value of the cosmological constant Λ_0 today in A units by

$$t_0 = (3/\Lambda_0)^{1/2} \tag{2}$$

and the cosmological constant is positive. This remarkable result means that while flat space-time quantum field theory still predicts $\Lambda_0 \neq 0$, the gravitational field equations admit a flat vacuum solution thus resolving the above dilemma. More significant is the fact that this allows a reinterpretation of the φ "field" of LNH. One conceives of φ as being a reflection of a time-dependent vacuum state. The energy of the vacuum state is decreasing relative to the energy of matter states. Hence the energy of matter states is increasing relative to the vacuum. This accounts for the various canonical LNH phenomena of accelerating particles, blue-shifting photons, and increasing particle number.

The reader unfamiliar with sign conventions, notation, or the units covariant formalism is referred to Paper I for details.

2. COSMOLOGICAL CONSTANT AND QUANTUM PHYSICS

During Einstein's original cosmological investigations he noticed that general relativity (without Λ) only allowed dynamically expanding or contracting isotropic, homogeneous Universes. Since the dominant philosophy of the day required a static Universe, Einstein (1917) noted that this could be achieved by the introduction of the cosmological constant Λ into his field equations. After Hubble's (1929) discovery of the linear relationship between galactic red shift and galactic distance it was realized that the natural interpretation of such observations was that the Universe is in a state of dynamical expansion. Had Einstein not bowed to philosophy he would have predicted the dynamical nature of the Universe several years before this was actually discovered. Einstein was so piqued by this result that he called the introduction of Λ "the biggest blunder of my life" (quoted in Misner et al., 1973, p. 410).

Unfortunately, many workers took this statement to mean that there are two theories of gravity, one being general relativity (Einstein's preferred theory) without Λ, and the other with Λ. This is false. The existence of Λ within the formal structure of general relativity is mandated by the logical structure of the theory itself. The *value* of Λ may well be zero as apparently believed by Einstein and advocated by others (Misner et al., 1973), but this is a matter to be determined by observation (Hickson and Adams, 1979) not philosophy.

Fifty years ago the cosmological constant was little more than a constant of integration in general relativity. The source of gravity is the energy tensor $T^{\mu\alpha}$. $T^{\mu\alpha}{}_{;\alpha}=0$. Hence the gravitational part of the gravitational field equations must also have zero divergence. The principle of equivalence requires that gravity be related to space-time geometry. There are exactly two geometrical tensors containing no higher than second derivatives with zero divergence: the Einstein tensor and the metric tensor. The simplest possible gravitational theory postulates a linear relationship among these three tensors so

$$G_{\mu\alpha}+\Lambda g_{\mu\alpha}+8\pi G T_{\mu\alpha}=0 \tag{3}$$

is the field equation for general relativity with Λ and G arbitrary constants to be fixed by observation. Observation requires G to be the Newtonian gravitational constant. Observation has not yet determined the value of Λ, although one can say (Misner et al., 1973, p. 411)

$$|\Lambda|/8\pi \lesssim 10^{-57}\ \mathrm{cm}^{-2} \tag{4}$$

Today one approaches theories of gravity from a different point of view, viz., gauge field theories. Central to a self-consistent formulation of gauge field theories of gravity is the Gauss–Bonnet–Cern theorem (Spivak, 1975). However, this theorem is valid only when applied to a compact group (Spivak, 1975). This means that the underlying gauge theory group for a gauge theory of gravity must be compact, a condition not satisfied by the Poincaré group. The simplest compact generalization of the Poincaré group is the de Sitter group and use of this group (MacDowell and Mansouri, 1977) automatically introduces the cosmological constant into the final theory.

Some workers have tried to avoid this by using the Poincaré group or groups containing the Poincaré group and arbitrarily requiring that certain "curvatures" vanish in order to obtain self-consistency (Kaku et al., 1977). However, the self-consistent way to incorporate such a restriction into a variational principle is through use of a Lagrange multiplier. When this is done the Lagrange multiplier becomes the cosmological constant. Consequently, while one could debate the inclusion of Λ in (3) on philosophical grounds 30 years ago, today one realizes that the theory itself requires Λ regardless of philosophy.

Zel'dovich (1968) has speculated that by taking the Λ term from the left-hand side of (3) to the right-hand side one can interpret Λ as being the ground-state energy of the vacuum. Hence setting $\Lambda=0$ is equivalent to asserting that the ground-state energy of the vacuum is zero. Since the vacuum by definition is empty, this seems to be a reasonable assumption. However, the vacuum is only empty in the sense that its contribution to $T^{\mu\alpha}$ is zero. It is quite possible that the vacuum state interacts with space-time through self-polarization effects so as to create an effective energy tensor $\Lambda g^{\mu\alpha}$ so far as the gravitational field is concerned. This would mean that even in empty space-time ($T^{\mu\alpha} = 0$) equation (3) reads

$$R_{\mu\alpha} = \Lambda g_{\mu\alpha} \tag{5}$$

and so does not admit Minkowski space-time as a vacuum solution.

This is one reason why constructive quantum field theory in a constant (nonzero) curvature space-time (Fronsdal, 1979) is significant because it allows one to compare results with quantum field theory in Minkowski space-time. If physically significant differences exist these can (in principle) be measured. This would allow one to determine via observation whether a system in a constant curvature space-time makes transitions among flat space-time quantum states, or whether the system is able to exist in fixed curved space-time quantum states.

From this point of view, anyone believing that quantum physics should be based on the structure of Minkowski space-time and not the structure of de Sitter space-time would prefer $\Lambda = 0$ in (5). Further, since the calculations leading to nonzero Λ were based on flat space-time quantum physics a question of self-consistency arises. Is it reasonable to use flat space-time quantum states to predict an effect which eliminates the possibility of a flat vacuum?

3. THE LNH FIELD

In previous papers (Adams, 1982, 1983) it was found that in the units covariant formalism all the LNH effects could be mediated through introduction of a measurable scalar "field" $\varphi(x)$. In A units the gravitational field equations became

$$^*G_{\mu\alpha} + \Lambda g_{\mu\alpha} + 8\pi G T_{\mu\alpha} = 0 \tag{6}$$

in place of (3) where

$$\Lambda = \Lambda_0(\varphi/\varphi_0)^2, \qquad G = G_0(\varphi_0/\varphi)^g \tag{7}$$

g is a constant parameter, and $^*G_{\mu\alpha}$ is the units covariant Einstein tensor with $\beta = \varphi$. In an empty space-time ($T^{\mu\alpha} = 0$) this becomes

$$G_{\mu\alpha} + 2\frac{\varphi_{;\mu\alpha}}{\varphi} - 4\frac{\varphi_{,\mu}\varphi_{,\alpha}}{\varphi^2} - g_{\mu\alpha}\left[2\frac{\Box\varphi}{\varphi} - g^{\lambda\rho}\frac{\varphi_{,\lambda}\varphi_{,\rho}}{\varphi^2} - \Lambda_0(\varphi/\varphi_0)^2\right] = 0 \quad (8)$$

Just as in the discussion of the cosmological constant in Section 2, those people wishing to base quantum physics on flat space-time quantum states would prefer to have $G_{\mu\alpha} = 0$ in (8). Just as in standard physics, calculations based on flat space-time quantum states can lead to a nonzero Λ. However, now there is a possible way out of this dilemma if one can find $\varphi(x)$ which cancels the effects of Λ in (8) with $G_{\mu\alpha} = 0$.

In the limit of empty flat space-time ($T^{\mu\alpha} \to 0$, $R^{\mu}{}_{\alpha\sigma\lambda} \to 0$) the Universe is still isotropic and homogeneous. Hence there exists a reference frame such that $\varphi = \varphi(t)$. Taking the (00) component of (8) with $G_{\mu\alpha} = 0$ gives

$$\left(\frac{\dot{\varphi}}{\varphi}\right)^2 = \frac{\Lambda_0}{3}\left(\frac{\varphi}{\varphi_0}\right)^2 \quad (9)$$

while taking the trace of (8) gives

$$2\frac{\ddot{\varphi}}{\varphi} - \frac{\dot{\varphi}^2}{\varphi^2} = \Lambda_0\left(\frac{\varphi}{\varphi_0}\right)^2 \quad (10)$$

The (ij) components of (8) are linear combinations of (9) and (10). Equation (9) immediately requires $\Lambda_0 \geqslant 0$. Hence

$$\frac{\dot{\varphi}}{\varphi^2} = \pm\left(\frac{\Lambda_0}{3\varphi_0{}^2}\right)^{1/2} \quad (11)$$

so the unique nontrivial ($\varphi \neq$ const) solution is

$$\varphi(t) = \pm\,\varphi_0(t_0/t) \quad (12a)$$

$$t_0 = (3/\Lambda_0)^{1/2} \quad (12b)$$

where the time origin was chosen so that $\varphi \to \pm\infty$ as $t \to 0$. Differentiating (11) and use of (9) shows that (10) is not an independent equation, so (12) is the general solution.

This is a remarkable result in that (choosing $\pm\,\varphi_0 > 0$) this is precisely the canonical LNH (Adams, 1983) value for $\varphi(x)$. This allows a reinterpretation of the meaning of φ. As emphasized by Canuto and Lee (1977), the calculation of Λ_0 from first principles, while beset with uncertainty, does lead to a relatively large value of Λ_0 for hot Universe models. They suggest that Λ_0 decreases with cosmic epoch due to the cooling of the Universe with expansion. The required time dependence is precisely given by (7) with (12). Consequently, the scalar "field" φ could be a compensating effect which is a macroscopic manifestation of a time- (temperature-) dependent vacuum state in spontaneous symmetry breaking gauge theories. The compensating φ "field" is necessary in order that quantum physics be based on flat space-time vacuum states, or alternatively, the fact that Nature is based on flat space-time quantum states induces a compensating φ "field". The cosmological constant Λ_0 is nonzero but its effect on space-time structure is compensated by φ.

4. SUMMARY AND DISCUSSION

The units covariant gravitational field equations containing LNH have been examined in A units in the limit of empty space-time ($T^{\mu\alpha} = 0$). It was shown that a flat space-time is admitted as a solution if and only if φ takes the canonical LNH form

$$\varphi = \varphi_0 (t_0/t) \tag{13a}$$

$$g = -1 \tag{13b}$$

with

$$t_0 = (3/\Lambda_0)^{1/2} \tag{13c}$$

where Λ_0 is the necessarily positive value of the cosmological constant today in A units.

This eliminates the dilemma existing in standard physics where flat space-time quantum physics leads to the prediction of a nonzero value for Λ so the empty space-time field equations do not admit a flat space-time solution. In this theory, in A units φ is a "compensating field" which allows an empty flat space-time solution even though a nonzero value for Λ is still predicted based on flat space-time quantum physics.

If φ is just a reflection of a time-dependent vacuum state this would explain why φ affects quantum dynamics but not quantum kinematics. The energy of the vacuum state is decreasing in time in A units. This shows up as a time-dependent Λ in A units. But by definition in A units quantum kinematics are not time dependent. One concludes that the energy difference between various matter quantum states and the vacuum state is increasing with time. Hence, particles speed up (Adams, 1982), photons get bluer (Adams, 1983) particle numbers increase (Adams, 1982, 1983). All these phenomena are manifestations of LNH and all can be "explained" in terms of the energy of a quantum system increasing in time relative to the vacuum state. The sole purpose of φ is to reflect this changing vacuum state.

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REFERENCES

Adams, P. J. (1982). *International Journal of Theoretical Physics*, **21**, 607.

Adams, P. J. (1983). *International Journal of Theoretical Physics*, **22**, 421.

Canuto, V., and Lee, J. F. (1977). *Physics Letters*, **72B**, 281.

DeWitt, B. S. (1975). *Physics Reports*, **19**, 295.

Dirac, P. A. M. (1937). *Nature*, **139**, 323.

Einstein, A. (1917). *Sitzungsberichte der Preussischen Akademie der Wissenschaften*, 142. Translated in *The Principle of Relativity*, Dover, New York, p. 175.

Fronsdal, C. (1979). *Physical Review D*, **20**, 848, and references therein.

Hickson, P., and Adams, P. J. (1979). *Astrophysical Journal Letters*, **234**, L87.

Hubble, E. P. (1929). *Proceedings of the National Academy of Sciences*, U.S.A., **15**, 169.

Kaku, M., Townsend, P. K., and van Nieuwenhuizen, P. (1977). *Physics Letters*, **69B**, 304.

Kirzhnitz, D. A., and Linde, A. D., (1976). *Annals of Physics*, (*N.Y.*) **101**, 195.

Linde, A. D. (1979). *Reports on Progress in Physics*, **42**, 389.

MacDowell, S. W., and Mansouri, F. (1977). *Physical Review Letters*, **38**, 739.

Misner, C., Thorne, K., and Wheeler, J. (1973). *Gravitation*, Freeman, San Francisco.

Spivak, M. (1979). *A Comprehensive Introduction to Differential Geometry*, Vol. 5, Publish or Perish, Inc., Berkeley, p. 385.

Zel'dovich, Ya. B. (1968). *Soviet Physics Uspekhi*, **11**, 381.